

# Meson Synchrotron Emission from Central Engines of Gamma-Ray Bursts with Strong Magnetic Fields

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## ABSTRACT

Gamma-ray bursts (GRBs) are presumed to be powered by still unknown central engines for the timescales in the range  $1ms \sim$  a few  $s$ . We propose that the GRB central engines would be a viable site for strong meson synchrotron emission if they were the compact astrophysical objects such as neutron stars or rotating black holes with extremely strong magnetic fields  $H \sim 10^{12} - 10^{17}G$  and if protons or heavy nuclei were accelerated to ultra-relativistic energies of order  $\sim 10^{12} - 10^{22}eV$ . We show that the charged scalar mesons like  $\pi^\pm$  and heavy vector mesons like  $\rho$ , which have several decay modes onto  $\pi^\pm$ , could be emitted with high intensity a thousand times larger than photons through strong couplings to ultra-relativistic nucleons. These meson synchrotron emission processes eventually produce a burst of very high-energy cosmic neutrinos with  $10^{12}eV \leq E_\nu$ . These neutrinos are to be detected during the early time duration of short GRBs.

*Subject headings:* cosmic rays — elementary particles — gamma rays: bursts — magnetic fields — stars: neutron

## 1. Introduction

Accumulating observational evidence shows the existence of astrophysical objects with extremely strong magnetic fields  $\sim 10^{15}G$ . Kouveliotou et al. (1998) have revealed that a galactic X-ray pulsar with an estimated magnetic field of  $\sim 8 \times 10^{14}G$  causes recurrent bursts of soft  $\gamma$ -rays, which are called soft gamma repeaters. Gamma-ray bursts (GRBs) with short and more intense bursts of  $100keV \sim 1MeV$  photons still remain puzzling although it has been progressively clearer that they are likely to have a cosmological origin. Some of the theoretical models of GRBs (Usov 1992, Kluźniak & Ruderman 1998, and Pacsyński 1998) invoke compact objects for their central engines such as neutron stars or rotating black holes with extremely strong magnetic fields  $H \sim 10^{16\sim 17}G$  in order to produce ultra-relativistic energy flow of a huge Lorentz factor  $\Gamma \sim 10^3$ . Furthermore, neutron stars or black holes with strong magnetic fields have been proposed by many authors (Hillas 1984, and references therein) as an acceleration site of ultra high-energy cosmic rays (UHECR). Milgrom & Usov (1995) suggested possible association of the two highest energy UHECRs with strong GRBs in their error boxes.

An ultra-relativistic nucleus gives rise to efficient meson emission, in analogy with the canonical photon synchrotron radiation, in such a strong magnetic field because it couples strongly to meson fields as well as to an electromagnetic field. Due to its large coupling constant the meson synchrotron emission is  $\sim 10^3$  times stronger than the usual photon synchrotron radiation. Ginzburg & Syrovatskii (1965a, b) calculated the intensity of  $\pi^0$  emission by a proton in a given magnetic field. At that time, however, they could hardly find the astrophysical sites to which their formulae could be applied.

In the present paper we propose that GRB central engines could be a viable site for strong meson emission. Waxman & Bahcall (1997) proposed that high energy neutrinos with  $\sim 10^{14}eV$  are produced by photomeson interactions on shock-accelerated protons in the relativistic fireball (Rees & Mészáros 1992). Pacsyński & Xu (1994) suggested that charged pions, which are produced in pp collisions when the kinetic energy of the fireball is dissipated through internal collisions within the ejecta, produce a burst of  $\sim 10^{10}eV$  neutrinos. Our proposed process of meson production is different from these two which can operate without magnetic fields. The very rapid variability time scale,  $\sim 0.1ms$ , of many GRBs implies that each sub-burst reflects the intrinsic primary energy release from the central engine (Sari & Piran 1997). Thus, the length scale of the central engine is estimated to be  $\sim 10km$ . The BATSE detector on board of Compton Gamma-Ray Observatory found the shortest burst of a duration of 5ms with substructure on scale of 0.2ms (Bhat et al. 1992). Accumulated BATSE data (Fishman & Meegan 1995) confirmed that

more than 25% of total events are short ( $\leq 2s$ ) bursts. Therefore, if the central engines of GRBs were the compact stellar objects like neutron stars or rotating black holes associated with strong magnetic fields, relativistic protons or heavy nuclei would trigger meson synchrotron emission whose decay products could provide several observational signals even before the hidden explosion energy is transported to the radiation in the later stage of a relativistic fireball.

We extend the previous treatment of  $\pi^0$  emission of Ginzburg & Syrovatskii (1965a, b) to several kinds of neutral and charged mesons, which couple to a nucleon through scalar or vector type interaction, in a manner somewhat different from theirs. If the produced mesons are  $\pi^0$ s or heavier mesons which have main decay modes onto  $\pi^0$ s, they produce high energy photons which are immediately converted to lower energy  $\gamma$ s or  $e^\pm$  pairs through  $\gamma \rightarrow \gamma + \gamma$  or  $\gamma \rightarrow e^+ + e^-$  (Eerber 1966) for their large optical depth in strong magnetic fields. However, if they are the charged mesons like  $\pi^\pm$ s, decay modes are very different and result in more interesting consequences. Since the source emitters of charged nuclei are accelerated to high energy (Hillas 1984) to some extent, very high energy neutrinos are produced by  $\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$  or its mirror conjugate process for the similar mechanism as proposed by Waxman & Bahcall (1997). These neutrinos could be a signature for strong meson emission from the GRB central engines and would be detected in the very early phase of the short GRBs.

We formulate the meson synchrotron emission in the next section, and the calculated results are shown and discussed in §3.

## 2. Semi-Classical Treatment of Meson Emission

We follow the established semi-classical treatment (Peskin & Schroeder 1995, Itzykson & Zuber 1980) of synchrotron emission of the quantum fields interacting with classical source current in strong external fields. In our present study the meson field is second quantized, while the nucleons are not and obey classical motion.

The energy spectrum of  $\pi^0$  synchrotron emission by a proton was first derived by Ginzburg & Syrovatskii (1965a, b) and it is give by

$$\frac{dI_\pi}{dE_\pi} = \frac{g^2}{\sqrt{3}\pi} \frac{E_\pi}{\hbar^2 c} \frac{1}{\gamma_p^2} \int_{y(x)}^\infty K_{1/3}(\eta) d\eta, \quad (1)$$

where  $g$  is the strong coupling constant,  $g^2/\hbar c \approx 14$  (Sakurai 1967),  $E_\pi$  is the energy of the emitted pion, and  $\gamma_p$  is the Lorentz factor,  $\gamma_p = E_p/m_p c^2$ , of the rotating proton in a given magnetic field.  $K_{1/3}$  is the

modified Bessel function of order  $1/3$ . The function  $y(x)$  is given by

$$y(x) = \frac{2}{3} \frac{m_\pi}{m_p} \frac{1}{\chi} x \left(1 + \frac{1}{x^2}\right)^{3/2}, \quad (2)$$

where  $m_p c^2 = 938 \text{ MeV}$  and  $m_\pi c^2 = 135 \text{ MeV}$  are the rest masses of proton and  $\pi^0$  meson, and the parameter  $\chi$ , which characterizes the synchrotron emission, is determined by the proton energy and the strength of magnetic field as  $\chi = \frac{H}{H_0} \gamma_p$  with  $H_0 \equiv \frac{m_p^2 c^3}{e \hbar} = 1.5 \times 10^{20} \text{ G}$ . In the above equations, variable  $x$  is introduced for mathematical simplicity,  $x = \frac{E_\pi}{E_p} \times \frac{m_p}{m_\pi}$ . Since the available energy of the pion satisfies  $m_\pi \leq E_\pi \leq E_p$ , the corresponding variable range of  $x$  is  $\gamma_p^{-1} \leq x \leq \frac{m_p}{m_\pi}$ .

Applying the same treatment to vector mesons, we extensively obtain the energy spectrum of  $\rho$  meson synchrotron emission

$$\frac{dI_\rho}{dE_\rho} = \frac{g^2}{\sqrt{3}\pi} \frac{E_\rho}{\hbar^2 c} \frac{1}{\gamma_p^2} \left(1 + \frac{1}{x^2}\right) \int_{y(x)}^\infty K_{5/3}(\eta) d\eta, \quad (3)$$

where  $K_{5/3}$  is the modified Bessel function of order  $5/3$ . The function  $y(x)$  is defined by Eq. (2) by replacing  $m_\pi$  with  $\rho$  meson mass  $m_\rho c^2 = 770 \text{ MeV}$ , and  $x = \frac{E_\rho}{E_p} \times \frac{m_p}{m_\rho}$ . Note that one can easily get the expression for photon synchrotron radiation in the limit of  $m_\rho \rightarrow 0$  by replacing the strong coupling constant  $g$  with the electromagnetic coupling constant  $e$ .

The total intensity of a scalar or vector meson as a function of  $\chi$  is obtained by integrating equation (1) or (3) over meson energies  $m_{\pi,\rho} \leq E_{\pi,\rho} \leq E_p$  or equivalently over  $\gamma_p^{-1} \leq x \leq \frac{m_p}{m_{\pi,\rho}}$ . It is useful to give an approximate formula of the total intensity in the limit of large or small  $\chi$ :

$$I_\pi = \begin{cases} \frac{g^2}{6} \frac{m_\pi^2 c^3}{\hbar^2}, & : \chi \gg 1 \\ \frac{g^2}{\sqrt{3}} \frac{m_\pi m_p c^3}{\hbar^2} \chi \exp\left(-\frac{\sqrt{3}}{\chi} \frac{m_\pi}{m_p}\right), & : \chi \ll 1 \end{cases} \quad (4)$$

and

$$I_\rho = \begin{cases} \frac{27\sqrt{3}}{16\pi} \Gamma(5/3) \frac{2g^2}{3} \frac{m_\rho^2 c^3}{\hbar^2} \chi^{2/3}, & : \chi \gg 1 \\ \frac{3}{2} \sqrt{\frac{3}{2}} \left(\frac{m_p}{m_\rho}\right) \left(1 + \left(\frac{m_\rho}{m_p}\right)^2\right)^{-1/4} \left(2 \left(\frac{m_\rho}{m_p}\right)^2 - 1\right)^{-1} \\ \times \frac{g^2}{\sqrt{3}} \frac{m_\rho m_p c^3}{\hbar^2} \chi^{3/2} \exp\left(-\frac{2}{3\chi} \left(1 + \left(\frac{m_\rho}{m_p}\right)^2\right)^{3/2}\right), & : \chi \ll 1 \end{cases} \quad (5)$$

where we have made an approximation  $K_\nu(\eta) \approx \frac{2^{\nu-1} \Gamma(\nu)}{\eta^\nu}$  for  $\eta \ll 1$  ( $\chi \gg 1$ ), or  $K_\nu(\eta) \approx \sqrt{\frac{\pi}{2\eta}} \exp(-\eta)$  for  $\eta \gg 1$  ( $\chi \ll 1$ ). These approximations are in reasonable agreement with exact numerical integrals within  $\pm 3\%$  for  $\pi$  meson and  $\pm 10\%$  for  $\rho$  meson at  $\chi \leq 0.01$  or  $10^2 \leq \chi$ .

Let us make a short remark on our classical treatment. When the proton energy is very high or the external magnetic field is strong, i.e.  $\chi \gg 1$ , the quantum effects not only in the meson field but in the

source nucleon current may not be negligible. In the case of photon synchrotron radiation, quantum effects were carefully studied (Erber 1966), and semi-classical treatment was found to be a good approximation to the exact solution within a few percent. In our treatment, the prefactor in Eq. (5)  $\left(\frac{27\sqrt{3}}{16\pi}\Gamma(5/3)\right) \approx 0.583$ . Taking the limit of  $m_\rho \rightarrow 0$  and  $g = e$ , Eq. (5) is applied to photon synchrotron radiation. It is shown (Erber 1966) that this factor should be 0.5563 in fully quantum mechanical calculation. It therefore is expected to hold true for the hadron processes as well.

### 3. Results and Discussions

Figure 1a shows a comparison between the calculated spectra of scalar  $\pi^0$  meson emission and  $\gamma$  synchrotron radiation. Since Usov (1992), Kluźniak & Ruderman (1998), and Pacsyński (1998) suggested strong magnetic fields of order  $H \sim 10^{16\sim 17}G$ , which are presumed to associate with neutron stars or black holes of the GRB central engines, we here take  $H = 1.5 \times 10^{16}G$ . The observed Lorentz factor of the fireball is  $\Gamma \sim 10^3$ , which indicates that the energy of charged particles is at least  $\sim 10^{12}eV$ . Although the acceleration mechanism in GRBs is still unknown, there are suggestions (Milgrom & Usov 1995) that the GRBs associate with UHECRs. Six events of UHECRs beyond the Greisen-Zatsepin-Kuz'min cutoff energy  $\sim 10^{20}eV$  (Hill & Schramm 1985) have been detected by AGASA group (Takeda et al. 1998). We therefore vary the proton energy from  $E_p = 10^{12}eV$  to  $10^{22}eV$ . All calculated spectra cut off sharply at incident proton energy  $E_{\pi,\gamma} = E_p$ . As seen in this figure, the intensity of  $\pi^0$  emission for  $E_p = 10^{12}, 10^{14}$  and  $10^{16}eV$ , which correspond respectively to  $\chi \approx 0.1, 10$  and  $1000$ , is  $10^3 \sim 10$  times stronger than that of  $\gamma$  radiation in high energy parts of the spectra. This reflects the different coupling constants,  $g^2/e^2 \sim 10^3$ . Very sharp declination of the  $\pi^0$  spectra at lower energy arises from the integral in Eq. (1) for finite pion mass because  $K_\nu(\eta) \approx \sqrt{\frac{\pi}{2\eta}} \exp(-\eta)$  for  $\eta \gg 1$ , which is a good approximation in this energy region where  $y(x) \gg 1$ .

Figure 1b shows comparison between the calculated spectra of  $\rho$  meson emission and  $\gamma$  synchrotron radiation. In this figure both spectra look very similar to each other, except for the absolute intensity and the sharp declination of low energy spectra due to finite  $\rho$  meson mass. This is because the interactions between  $\rho$  meson and proton and between photon and proton are of vector type. For the proton energies above  $\sim 10^{14}eV$  the intensity of  $\rho$  meson emission is roughly a thousand times larger than that of  $\gamma$  synchrotron radiation, reflecting again its stronger coupling constant.

Figure 2 displays the calculated total intensities of synchrotron emission of  $\pi^0$  and  $\rho$  mesons as a function

of  $\chi$ . These are the integrated spectra shown in Figs. 1a & 1b over available meson energies. The total intensity of  $\gamma$  synchrotron radiation is also shown in this figure.  $I_{\pi^0}$  exceeds  $I_\gamma$  at  $3 \times 10^{-2} < \chi < 3 \times 10^4$ , and  $I_\rho$  exceeds  $I_\gamma$  at  $\chi > 0.2$ . Both  $I_{\pi^0}$  and  $I_\rho$  decrease exponentially with decreasing  $\chi$  due to their finite masses. (See the asymptotic forms at  $\chi \ll 1$  in Eqs. (4) and (5).) This sharp declination of  $I_\rho$  takes place at higher  $\chi$  than  $I_{\pi^0}$  because  $\rho$  meson mass is larger than pion mass.  $I_\rho$  resembles  $I_\gamma$  at  $\chi > 1$ , except that the intensity is a thousand times different from each other, reflecting that both  $\rho$  meson and photon have the same vector type coupling to proton with different coupling constants,  $g^2/e^2 \sim 10^3$ .

A nucleon strongly couples to  $\pi^\pm$  as well as  $\pi^0$  field. The interaction Hamiltonian is charge independent and  $H_{\text{int}} = ig(\sqrt{2}\bar{\psi}_n\gamma_5\psi_p\phi_{\pi^+} + \bar{\psi}_p\gamma_5\psi_n\phi_{\pi^0}) + c.c.$  (Sakurai 1967). The initial state  $\psi_p$  in both  $p \rightarrow n + \pi^+$  and  $p \rightarrow p + \pi^0$  processes is identical. The difference comes from the final states: Proton and  $\pi^+$  couple to external magnetic field, but neutron and  $\pi^0$  do not. (Neutron has too small magnetic moment.) When we describe these processes in quantum mechanics, we should use wave functions from the solution of Dirac and Klein-Gordon equations for the nucleon and the pion separately. However, final states are more or less the same for the same charge state, aside from the different masses. These might not change the reaction amplitudes by many orders at ultra-relativistic energies where the rest mass is neglected. Note that the conjugate processes  $n \rightarrow p + \pi^-$  and  $n \rightarrow n + \pi^0$  can also occur when a composite nucleus like  $^{56}\text{Fe}$  orbits in the strong magnetic fields.

Heavy mesons including  $\rho$  meson have several appreciable branching ratios for the decay onto  $\pi^\pm$ . Let us discuss what kinds of observational signals they may make.

We assume that some fraction of  $10^{51} \sim 10^{53} \text{ ergs}$  of the gravitational or magnetic field energy is released by some unknown mechanism operating at the GRB central engine during very short time duration,  $\sim 0.1 \text{ ms}$ , of the first sub-burst. We also assume that an appreciable part of this energy is deposited into the relativistic motion of the material leading to UHECRs. In a somewhat different context, Waxman & Bahcall (1997) proposed that the photomeson production in the ejecta of the fireball would make a burst of  $\sim 10^{14} \text{ eV}$  neutrinos. Although their proposed mechanism of meson production is completely different from ours, we can apply similar discussion on the physical consequence of  $\pi^\pm$  decay. Extending our previous discussions of  $\pi^0$  in Fig. 2 to  $\pi^\pm$ , we expect that a thousand times stronger intensity of high energy neutrinos than the photons can be produced universally at  $0.1 \leq \chi$  from  $\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$  and its mirror conjugate process. Since neutrinos can escape from the ambient matter, these neutrinos could be a clear signature showing strong meson synchrotron emission near the central engines of GRBs associated with extremely strong magnetic fields.

The neutron emerging from  $p \rightarrow n + \pi^+$  inherits almost all proton energy and can escape from the region of strong magnetic field. If there is not a dense shell surrounding the central engine, it travels  $\sim 10^5 \times (\gamma_n/10^{10}) pc$  before beta decay. This process may also produce a very high energy neutrino. The generic picture of GRBs (Mészáros & Rees 1993, Piran 1999) suggests that a baryon mass  $\sim 10^{-5} M_\odot$  is involved in a single explosion. If this is the case, such amount of baryon mass is huge enough to stop almost all neutrons before running through the ambient matter.

If the central engines are neutron stars or black holes, the material ejected from these compact objects contains heavy nuclei such as oxygen and iron because these are the products from evolved massive stars. Therefore, meson emission from a heavy nucleus as well as from a proton is worth being considered. Let us consider a nucleus of total energy  $E_{\text{tot}}$ , mass number  $A$ , and charge  $Z$ , in a magnetic field of strength  $H$ . The energy of each nucleon is  $E = E_{\text{tot}}/A$ . When the strength of the effective magnetic field is  $H_{\text{eff}} = \frac{Z}{A}H$ , the orbital trajectory of a proton is the same as the trajectory of the nucleus in the magnetic field  $H$ . The intensity of  $\pi^0$  emission by the nucleus should be the sum of each nucleonic contribution provided that the synchrotron emission is incoherent. Thus the total intensity is given by

$$I_{\pi^0}^{(A)}(E_{\text{tot}}, H) \approx A \times I_{\pi^0}^{(p)}(E, H_{\text{eff}}), \quad (6)$$

where  $I_{\pi^0}^{(p)}(E, H_{\text{eff}})$  is the intensity of  $\pi^0$  emission by the nucleon of energy  $E$  in magnetic field  $H_{\text{eff}}$ . Note that both proton and neutron emit  $\pi^0$ . Figure 3 shows  $I_{\pi^0}^{(56)}(E_{\text{tot}}, H)$  for  $^{56}\text{Fe}$  as a function of total energy  $E_{\text{tot}}$  with a fixed magnetic field of  $H = 1.5 \times 10^{12} G$ . The sharp declination of  $I_{\pi^0}^{(56)}$  takes place at higher energy than  $I_{\pi^0}^{(p)}$  because each nucleon in  $^{56}\text{Fe}$  has effectively smaller energy than a single proton.

There is now more motivation to study the GRBs in association with UHECRs. It is highly desirable to proceed with a project like Orbiting Wide-angle Light-collectors (OWL), in order to detect ultra-relativistic neutrinos from GRBs for finding the true nature of the central engines.

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#### 5. Figure Captions

Figure 1: (a) Calculated energy spectra of scalar  $\pi^0$  meson emission (solid curve) and photon synchrotron radiation (dashed curve) for various incident proton energies  $E_p$  with a fixed magnetic field of  $H = 1.5 \times 10^{16} G$ . Denoted numbers in the figure are the proton energies  $E_p = 10^{12}, 10^{14}, 10^{16}, 10^{18}, 10^{20}$ ,

and  $10^{22}eV$  from left to right. (b) The same as those in (a) for vector  $\rho$  meson emission.

Figure 2: Calculated total intensities of the emission of scalar  $\pi^0$  meson (solid curve), vector  $\rho$  meson (long-dashed curve), and photon  $\gamma$  (dashed curve) as a function of  $\chi = \frac{H}{H_0}\gamma_p$ .

Figure 3: Calculated total intensities of scalar  $\pi^0$  meson emission by the proton (dashed curve) and iron nucleus (solid curve) as a function of the total energy  $E_{tot}$  with a fixed magnetic field of  $H = 1.5 \times 10^{12}G$ .